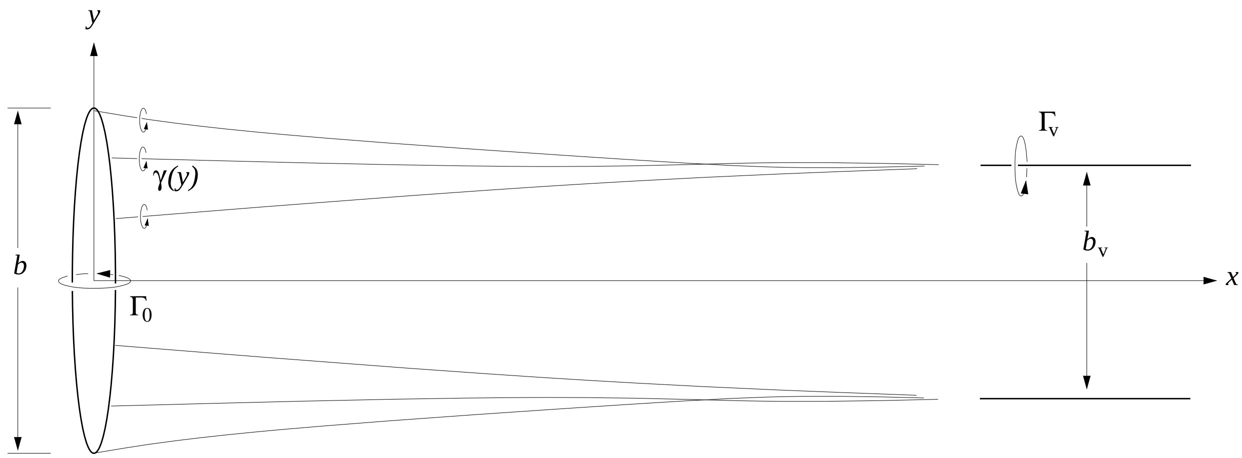
Unified Engineering Spring 2008
Fluids Problem 7

Although our wing mathematical model assumes that the wing's vortices trail straight downstream, in reality they *roll up* into two concentrated tip vortices, which have a somewhat smaller span b_v than the wing's geometric span b. See Figure 5.5 in Anderson (for a higher-aspect ratio, the rollup is more gradual as sketched here).



It is known that during the roll-up process, the so-called moment of vorticity cannot change. This is defined as follows, for a sheet and for discrete vortices.

$$I_v \equiv \int_{-b/2}^{b/2} \gamma \, y \, dy \qquad \text{(vortex sheet)}$$

$$I_v \equiv \sum \Gamma_v \, y_v \qquad \text{(discrete vortices)}$$

We will assume that the wing has an elliptic circulation distribution

$$\Gamma(y) = \Gamma_0 \sqrt{1 - (2y/b)^2}$$

which means that the rolled-up tip vortices have a circulation of

$$\Gamma_v = \Gamma_0$$

a) Evaluate I_v just behind the trailing edge.

b) Evaluate I_v in the rolled-up wake.

c) Using the fact that I_v cannot change, determine the rolled-vortex span b_v. Check by eyeball against Figure 5.5 in Anderson.